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Optimal location of heat sources on a vertical wall with natural convection through genetic algorithms

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Abstract

Passive cooling of electronic components by natural convection heat transfer is the least expensive, quietest and most reliable method of heat rejection. However, some characteristics intrinsic to the phenomenon, such as a non-linear interaction between heat sources difficult its practical application. The objective of this work is to study the natural convection heat transfer optimization through genetic algorithms. Four cases were studied: one heat source, two heat sources with same dissipation rate, and two heat sources with different dissipation rates on a vertical wall. The results showed good agreement with literature and confirmed the methodology as computationally feasible for the optimal location of heat sources in a vertical wall.

Keywords: Natural convection; Optimal location; Genetic algorithms

1. Introduction

Natural convection heat transfer plays an important role in the electronic components cooling since it has desirable characteristics in thermal equipments design: absence of mechanical or electromagnetic noise; low energy consumption, very important in portable computers; and reliability, since it has no elements to fail.

The optimization of the heat transfer has increasingly importance in electronic packaging due to the higher heat densities and to the electronic components and equipments miniaturization [1]. In spite of the abundant results about natural convection in electronic packaging, works dealing with heat transfer maximization is scarce in literature [2–4]. This is, probably, due to the non-linear nature and to the difficult of natural convection simulation.

In many numerical and experimental works, heat source location is studied, especially with two or more sources on a vertical wall. The parameters frequently considered are: the Rayleigh number, the distance between heat sources and the ratio between heat source dissipation rates [2,5–7].

Recently, da Silva et al. [4] studied the optimal heat source distribution on an adiabatic wall. They carried out a theoretical analysis and numerical simulations of the natural convection in a vertical cavity. In the work, curves showing the heat source optimal location as function of Rayleigh number are presented for cavities with one, two and three heat sources. To obtain the optimal locations they extensive simulated all combinations between the control parameters, Rayleigh number and heat source locations. They concluded, contrary to the results presented by Liu and Phan-Thien [2], that the optimal arrangement is not described by a constant ratio between the center-to-center distance between heat sources, but by a function that depends strongly on the Rayleigh number and the heat source dimension.

In this work, we carried out the optimization of the natural convection heat transfer in a two-dimensional vertical square cavity with one or two heat sources mounted on an adiabatic wall. The main goal here was to check the computational feasibility of the genetic algorithm method as

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Nomenclature

C	global conductance	*Greek symbols*	
c_p	specific heat	α	thermal diffusivity
D	heat source dimension	β	thermal expansion coefficient
g	gravitational acceleration	ν	kinematics viscosity
H	cavity height	μ	viscosity
k	thermal conductivity	ρ	density
P	pressure		
Pr	Prandtl number, ν/α	*Subscripts*	
q''	heat flux	0	at lower source
R	dissipation ratio, $q_0''/(q_0''+q_1'')$	1	at upper source
Ra^*	modified Rayleigh number, Eq. (9)	∞	ambient value
S	heat source location	max	maximum at the heat source
T	temperature		
u	horizontal velocity component	*Superscript*	
v	vertical velocity component	$\sim$	dimensionless value
x, y	spatial coordinates		

a promising method to optimize heat transfer in electronic packaging.

2. Analysis

2.1. Problem formulation

In order to compare the results, the problem formulation followed that proposed in [4]. The flow was assumed incompressible and laminar with constant fluid properties except for the density change with temperature in the buoyancy term. The finite volume control method was used for the discretization of the elliptic governing equations. The steady state two-dimensional versions of continuity, momentum and energy equations are:

Continuity

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

Momentum equations

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial P}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \tag{2}$$

$$\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial P}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)+\rho g\beta(T-T_\infty) \tag{3}$$

Energy equation

$$\rho c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \tag{4}$$

where T_∞ is the temperature at the vertical opposing wall.

The equations are made dimensionless using the characteristic length of the vertical cavity H, the thermal diffusivity α, the density ρ, and the uniform heat flux q_0'' prescribed at the lower heat source as reference quantities. The dimensionless variables are

$$\tilde{x}=\frac{x}{H},\quad \tilde{y}=\frac{y}{H} \tag{5}$$

$$\tilde{u}=\frac{u}{(\alpha/H)},\quad \tilde{v}=\frac{v}{(\alpha/H)} \tag{6}$$

$$\widetilde{P}=\frac{P}{\rho(\alpha/H)^2} \tag{7}$$

$$\widetilde{T}=\frac{(T-T_\infty)}{q_0''H/k} \tag{8}$$

The modified Rayleigh number was defined as

$$Ra^*=\frac{g\beta q_0''H^4}{\alpha\nu k} \tag{9}$$

where g is gravitational acceleration, k is the thermal conductivity, β is the volumetric coefficient of thermal expansion and ν is the kinematic viscosity.

Therefore, the dimensionless form of the governing equations is

Continuity

$$\frac{\partial \tilde{u}}{\partial \tilde{x}}+\frac{\partial \tilde{v}}{\partial \tilde{y}}=0 \tag{10}$$

Momentum equations

$$\tilde{u}\frac{\partial \tilde{u}}{\partial \tilde{x}}+\tilde{v}\frac{\partial \tilde{u}}{\partial \tilde{y}}=-\frac{\partial \widetilde{P}}{\partial \tilde{x}}+Pr\left(\frac{\partial^2 \tilde{u}}{\partial \tilde{x}^2}+\frac{\partial^2 \tilde{u}}{\partial \tilde{y}^2}\right) \tag{11}$$

$$\tilde{u}\frac{\partial \tilde{v}}{\partial \tilde{x}}+\tilde{v}\frac{\partial \tilde{v}}{\partial \tilde{y}}=-\frac{\partial \widetilde{P}}{\partial \tilde{y}}+Pr\left(\frac{\partial^2 \tilde{v}}{\partial \tilde{x}^2}+\frac{\partial^2 \tilde{v}}{\partial \tilde{y}^2}\right)+Ra^*Pr\widetilde{T} \tag{12}$$

Energy equation

$$\tilde{u}\frac{\partial \widetilde{T}}{\partial \tilde{x}}+\tilde{v}\frac{\partial \widetilde{T}}{\partial \tilde{y}}=\left(\frac{\partial^2 \widetilde{T}}{\partial \tilde{x}^2}+\frac{\partial^2 \widetilde{T}}{\partial \tilde{y}^2}\right) \tag{13}$$

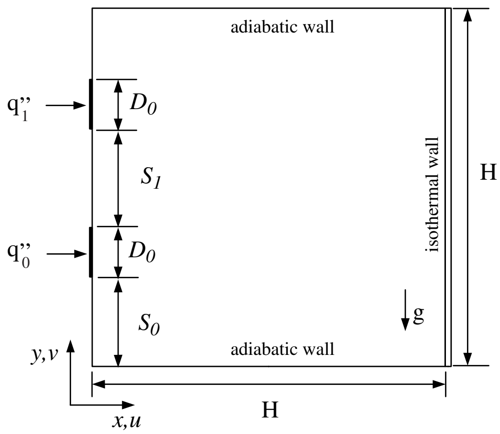

Fig. 1. Problem definition.

The computational domain was a square cavity with all surfaces assumed adiabatic, except for the isothermal opposing wall, and with no-slip condition. As it can be seen in Fig. 1, the boundary conditions were established to simulate discrete heat sources flush mounted on an adiabatic vertical wall as in Ref. [4].

2.2. Numerical procedure

Numerical solution was obtained by finite volumes method using the SIMPLE algorithm [8], since the flow was assumed incompressible. The interpolation of gradients of velocities and temperatures used the UPWIND algorithm. Non-uniform unstructured triangular mesh was used with smaller volumes close to the vertical walls. For each studied case, in the range of Ra from 10^2 to 10^6 and $Pr = 0.7$, tests were performed to check the independence of results on the mesh refinement (e.g. Fig. 2). The resulting 10,000 volumes mesh was employed in all cases. The algebraic system resulting from numerical discretization was solved by TDMA (Tridiagonal Matrix Algorithm) applied in a line going through all volumes in the computational domain. Validation of the numerical procedure was checked by solving the problem presented by Davis [9].

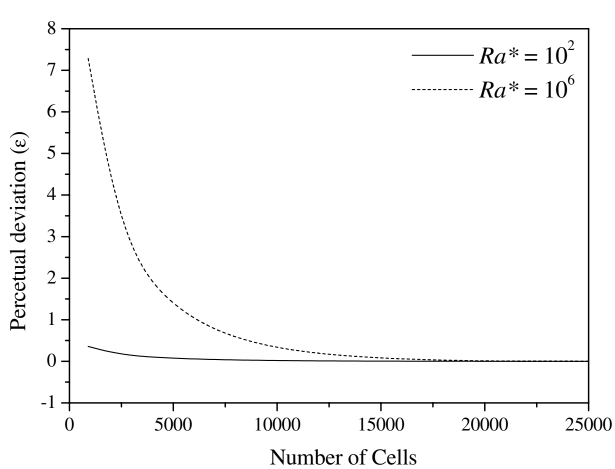

Fig. 2. Convergence of the percent deviation of maximal temperature in the cavity with number of cells.

2.3. Genetic algorithms

A genetic algorithm is a method for rapidly and efficiently searching through the space of all possible solutions to a given problem. It is well suited for cases where gradient information is not available or is computationally expensive.

The basic idea of the genetic algorithm is to consider an initial randomly generated population (ensemble) of possible solutions. The fitness of each individual, or parameters set, is represented by the objective function, or in this work by a numerical simulation for the parameters set. The individuals better fitted are selected to randomly mate (crossover). The new individuals and some of the best individuals from the last generation constitute the new generation [10].

The fitness evaluation and the creation of new generations are repeated until an adequate solution is found. Genetic algorithms have the advantage of searching multiple areas in solution space in parallel, which often prevents focusing on a local solution and make easy the use of parallel computation with workstations clusters. This method has been used in several engineering applications dealing with multi-parametric optimization [11–20].

In this work, the individual is represented by a case with a set of codified chromosomes (heat sources locations). The fitness value is represented by the global conductance as defined in Ref. [4],

$$C = \frac{D_0(q_0'' + q_1'')}{k(T_{\max} - T_\infty)} \tag{14}$$

To calculate the population fitness values, a numerical simulation must be performed, until convergence, with the corresponding set of heat sources locations for each individual. Since several individuals and populations must be evaluated, the computational cost may become prohibitive. Due to this, we used the Micro-Genetic Algorithm (μGA) [21], since it uses a reduced population size, and therefore less fitness function evaluations. The μGA can be described as follows:

(1) A small population (five individuals) is randomly generated.
(2) The fitness value for each individual is obtained and the best fitted is 'cloned' to the next generation (elitism).
(3) The individuals are chosen to mate by a roulette, in which each compartment is proportional to the fitness value magnitude. The best fitted mate more often.
(4) The uniformity of population is checked. If it is uniform, the best one is kept, the others are substituted by new randomly generated individuals in the step 1; otherwise go back to step 2.

It can be noticed that mutations are not used in the μGA, since the diversity, necessary to the good working of the algorithm, is introduced if the individuals of a given generation are very similar, that is, the population reaches the uniformity.

The GAlib, a C++ genetic algorithm library developed by Wall [22] at Massachusetts Institute of Technology, was used to implement the Micro-Genetic Algorithm.

3. Results

The integration between the optimization and the simulation softwares was carried out through command line. The optimization software called the simulation software with command line parameters defining the heat sources locations. This made the use of parallel computation straightforward, since the optimization software could ask for any node of a workstation cluster to execute the numerical simulation.

The first case studied was a square cavity with a heat source of length $s = 0.1$. da Silva et al. [4] obtained the optimal location through an extensively search in the range 0–1. It is not mentioned by da Silva et al. [4] the number of numerical simulations performed, but since the used grid had 101 elements in the vertical direction for the source dimension of 0.1, it was presumed that about 90 simulations for an extensively optimal search in this case were needed.

A good agreement was found between our results, for $Ra^* = 10^3$ and that of the literature can be seen in Fig. 3. Each point plotted is a heat source location used during the optimization process, and for this case 45 numerical simulations were performed, half of those likely to have been performed in an extensive search. The optimal heat source location found was $s_{\text{opt}} = 0.45$, the same value obtained by da Silva et al. [4].

Fig. 4 shows the optimal heat source locations as a function of the modified Rayleigh number. It can be noticed the good agreement with the literature data. The deviation

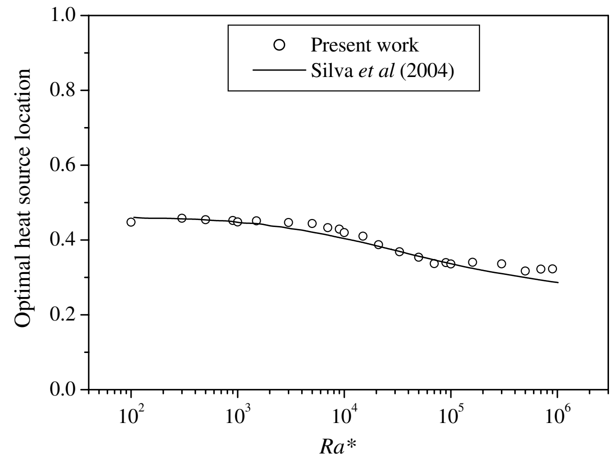

Fig. 4. Optimal location of a single heat source.

from the literature data is due to a limitation of genetic algorithm employed. It has stronger global convergence than local convergence.

As the average fitness of the population approximates the optimal value, the convergence rate becomes slower. There are many methods to increase the convergence rate near the optimal value, increasing the local precision [17]. However, this is not the scope of this work, since this work is a preliminarily study on the use of the genetic algorithm in natural convection. The optimal location of electronic components in an industrial application would depend on many parameters and restrictions so that small deviations from the global optimal locations are not very important.

The second case studied was the vertical cavity with two heat sources with the same dissipation rate. This case was also studied by da Silva et al. [4], and it is presumed that about 8000 numerical simulations were necessary in an extensive search by the optimal heat sources locations. In this work, about 160 simulations were performed for each value of the modified Rayleigh number value used. The results are presented in Figs. 5 and 6, where it can be

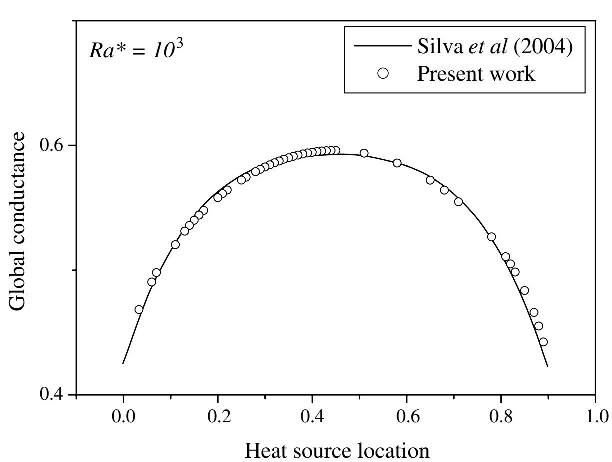

Fig. 3. Global conductance as a function of the heat source location.

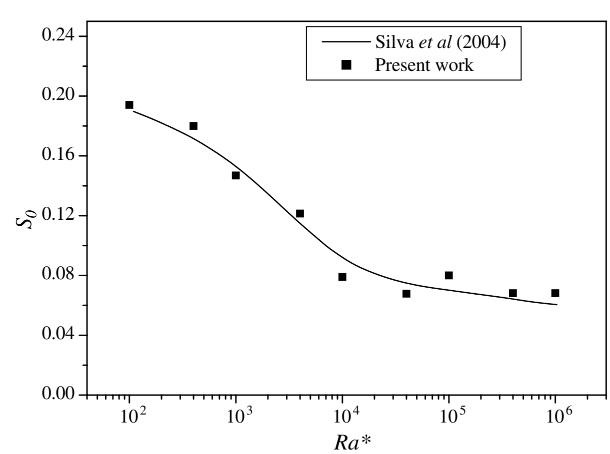

Fig. 5. Optimal location of the lower heat source as a function of the modified Rayleigh number.

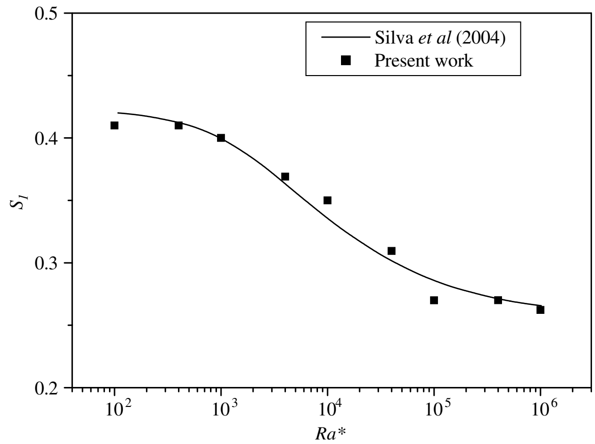

Fig. 6. Optimal location of the upper heat source as a function of the modified Rayleigh number.

noticed the good agreement with literature and the small deviations due to the local limitation of the optimization algorithm employed. Figs. 5 and 6 show the optimal locations of the lower and upper sources, respectively, as functions of the modified Rayleigh number.

For the third configuration, two cases were studied with different dissipation rates in the heat sources. One case where the larger heat dissipation was in the upper heat source and another where the higher dissipation was in the lower heat source. In both cases the magnitude of the larger dissipation rate was twice the value of the other one. For $Ra^* = 10^6$, the optimization software found that the better configuration, for both cases initially defined, is that where the larger heating rate source is located at $s_0 = 0.29$ from the bottom cavity and $s_1 = 0.43$ from the upper heat source. These cases were performed to check if the optimization software was able to find the better heat source configuration and location according to the heat dissipation rate, independently of the initial configurations provided to the software.

As an example of a problem with three parameters of optimization, in the last case studied, the location of the two heat sources and the ratio between the heat generated by one of them and that generated by both (R) were optimized for $Ra^* = 10^2$ and $Ra^* = 10^6$. For $Ra^* = 10^2$, the optimum arrangement found was $s_0 = 0.19$, $s_1 = 0.41$ and $R = 0.5$, this is $q_0'' = q_1''$, with about 380 simulations performed.

For $Ra^* = 10^6$, the optimum location found was that where the larger heating rate source is located at 0.21 from the bottom cavity and the smaller heating rate source is located at 0.43 from the bottom cavity and $R = 0.008$. It can be seen in Fig. 7 that the maximum global conductance is not dependent on the location of the smaller heating rate source. In this case, the optimal ratio R is small, tending to zero, which means that the larger source is several times bigger than the smaller one. Then, its influence is relatively too small, which makes its location to be not important in this case. Fig. 8 shows the global conductance as a function of the distance of the larger heat source from the bottom cavity, and it can be seen that the maximum point occurs around the 0.2 distance. Fig. 9 shows the

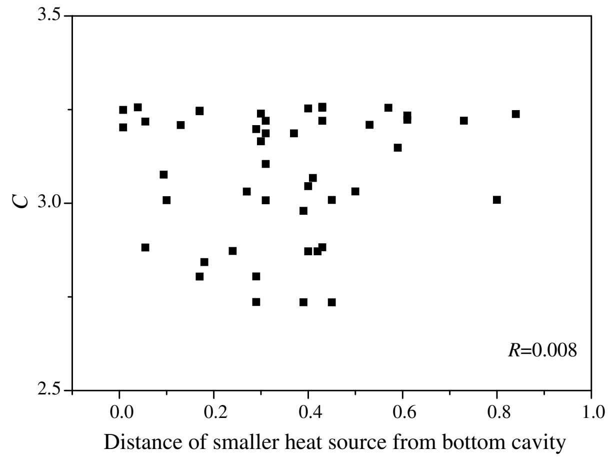

Fig. 7. Global conductance as a function of the distance of smaller heat source from bottom cavity ($R = 0.008$).

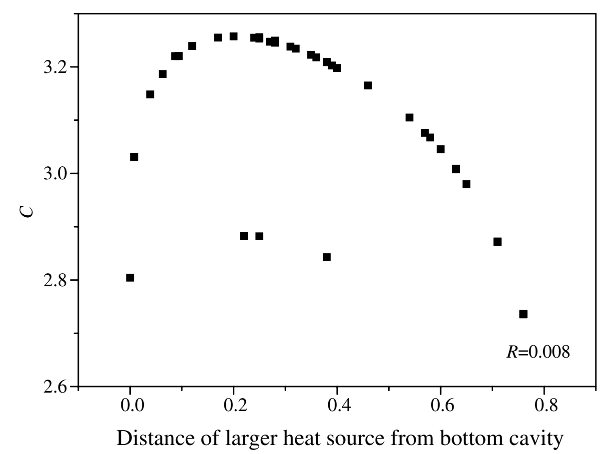

Fig. 8. Global conductance as a function of the distance of larger heat source from bottom cavity ($R = 0.008$).

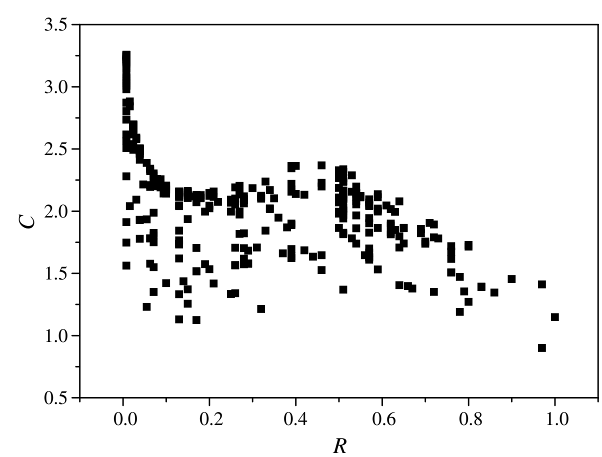

Fig. 9. Global conductance as a function of the ratio R.

global conductance as a function of the ratio R, it can be noticed a local maximum point around $R = 0.5$, where both heat sources have the same dissipation, as observed for $Ra^* = 10^2$.

4. Conclusions

Natural convection heat transfer is often the preferred method of cooling electronic devices. When thermal sources are in close proximity to each other on a surface they mutually interact. This non-linear interaction between the heat sources, together with low heat transfer coefficients makes it difficult in practice to use this heat transfer mode in the electronic industry.

In this work, the optimization of the natural convection heat transfer was carried out for a two-dimensional vertical square cavity with one or two heat sources mounted on an adiabatic wall. A Micro-Genetic Algorithm was employed as the optimization method where the numerical simulation by finite volumes represented the objective function. For each population generation, the fitness evaluations, or numerical simulations, were done in a workstation cluster, and since the numerical simulations were independent of each other the whole processing time was divided by the number of nodes in the workstation cluster.

The cases studied here were chosen in order to compare the results with those presented by da Silva et al. [4]. The first and the second cases studied considered a square cavity with one and two heat sources, respectively. Good agreements between the obtained results and those presented by da Silva et al. [4] were found. For the first case about 50 numerical simulations were performed for each value of the Rayleigh number used, approximately half of those that would be performed in an extensive search. A number of 160 simulations were performed for the second case, which represents about 2% of the numerical simulations that would be necessary in an extensive search.

Small deviation between our results and the literature data was found due to a slow local convergence rate, which is a characteristic of the genetic algorithms. Since this work is a preliminarily study on the use of the genetic algorithm in electronic packaging, no improvements were attempted in the algorithm to overcome this limitation.

The third configurations were two cases with two heat sources with different dissipation rates. One case where the larger heat dissipation was in the upper heat source and another where the higher dissipation was in the lower heat source. These cases were used to show that the optimization software was able to find the better heat source configuration and location according to the heat dissipation rate, independently of the initial configurations provided to the software.

In the last case studied, the location of two heat sources and the ratio between the heat generated by one of them and that generated by both (R) were optimized for $Ra^* = 10^2$ and $Ra^* = 10^6$. It was found that for $Ra^* = 10^2$, the maximum global conductance occurred at $s_0 = 0.19$, $s_1 = 0.41$ and $R = 0.5$, this is $q''_0 = q''_1$. And for $Ra^* = 10^6$, the maximum global conductance occurred when the larger heating rate source was located at 0.21 from the bottom cavity, the smaller heating rate source was located at 0.43 from the bottom cavity and $R = 0.008$. It was pointed out the local maximum point, around $R = 0.5$, in the global conductance behavior, as observed for $Ra^* = 10^2$. This suggests a transition between the ratio R where the maximum global conductance occurs as a function of the modified Rayleigh number. This behavior will be studied in depth in a future work.

In this work, we showed that the genetic algorithm is a computationally feasible optimization methodology for the optimal location of heat sources in a vertical wall.

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